

**NOTES ON COST AND COST ESTIMATION**  
by  
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The basic unit of the cost analysis is the flight segment. In describing the carrier's cost we distinguish costs which vary by segment and those which vary by route. In many cases the source of the difference in costs will be in the airline system or station (airport costs). For example, if carrier J were to extend its operation from point B to point C, in an AB market, the additional costs would be increased by the flight operating costs and some passenger costs but since it was already using the airport at B, the cost of adding operations from this station may be relatively small.

Since costs are different between fareclasses, it is necessary to estimate the cost of each fareclass by carrier and by flight segment. However, in the long run carriers allocate total usable space in a plane between fareclasses (First, Business and Economy seats) in such a way to equalize the marginal revenues per squarefoot for all fare categories. Therefore, given knowledge of the physical space required to put a seat of each class and the optimal load factors for each fare class, it is possible to convert passengers of all classes into a scaler (standard class equivalent) for costing purpose. For example, for a carrier-segment combination (henceforth referred to simply as 'segment') the passenger volume can be scalarized as:

$$Y = a_1 Y_1 + a_2 Y_2 + Y_3 \quad (1)$$

where  $Y$  is the total passenger volume (standard class equivalent),  $Y_i$  is number of *ith* fareclass passengers, and  $a_i$  is the conversion factor for *ith* fareclass to the standard fare class equivalent. The sizes of  $a_1$  and  $a_2$  can be determined by the procedure explained in Oum, Gillen and Noble (1985).<sup>1</sup> The unit cost of  $Y_1$  is  $a_1$  times the unit cost of  $Y$ .

In order to compute a carrier's total cost on a segment, say  $S$ , it is necessary to identify the total traffic volume using that segment by aggregating all O-D traffic travelling via segment  $S$  as follows:

$$Y^S = \sum_{OD} \sum_{r \in S} q_r^{OD} \quad (2)$$

where  $Y^S$  is the total traffic volume on segment  $S$ , and  $q_r^{OD}$  is the O-D demand volume choosing route  $r$  and is computed from the demand model.

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<sup>1</sup> See T. Oum, D. Gillen and D. Noble, "Demands for Fareclasses and Pricing in Airline Markets" Logistics and Transportation Review, Vol. 23, (1986)

## (B) Measurement of Segment Cost

The segment cost has two components: the costs which vary with passengers and flights, and those which remain unchanged. The latter consists of some portion of airport costs and the indirect costs of the carrier to be allocated to the segment. Therefore, the segment cost (C) can be written as:

$$C = A_s + C^s(Y, F) \quad (3)$$

where  $A_s$  is the segment cost which does not vary with passenger volume (Y) or flight frequency (F), and  $C^s(Y, F)$  is the segment cost which varies with Y and F. Two simply alternative specifications for the segment cost function are represented in 3a and 3b.

$$C(\bullet) = A_s + b_1 Y + b_2 F + b_3 Y F \quad (3a)$$

$$C(\bullet) = a Y^{b_1} F^{b_2} \quad (3b)$$

The total variable cost of segment S,  $c^s(Y, F)$ , consists of two components: the costs related to operating aircraft, and the costs associated with passenger handling at the airports and a portion of indirect and administration costs related to number of passengers and flight frequency.<sup>2</sup>

The cost associated with operating aircraft on a segment (henceforth referred to as flying operations cost (FOC)). FOC can be measured by adding the cost per block hour multiplied by the number of block hours required for the flight segment:

$$FOC_s = B_s \cdot H_s \cdot f(Y_s) \quad (4)$$

where  $B_s$  is cost per block hour for the aircraft used,  $H_s$  are the block hours required for segment S, and  $f(Y_s)$  is flight frequency which depends on number of passengers on segment,  $Y_s$ . Obviously, the cost per block hour for a given aircraft and given segment will depend upon the labour prices of the carrier as well as other input prices. However, the other input prices such as fuel, maintenance and capital costs will not

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<sup>2</sup>. The aircraft cost can be measured by adding the cost per block hour multiplied by the number of block hours required for the flight segment and a portion of the indirect airline costs which are attributable flight frequency (how to get this?). Note that the block-hour costs need to be adjusted upward by the amount of interest cost on the capital tied up in aircraft. It appears that the cost per block hour available in Form 41 data includes only the aircraft rentals paid for leased aircraft, and does not appear to include the interest cost on the owned aircraft. This may not be all that important.

vary significantly across carriers for that segment. Therefore, FOC for a carrier can be parameterized as a function of its' labour cost relative to a base carrier as follows:

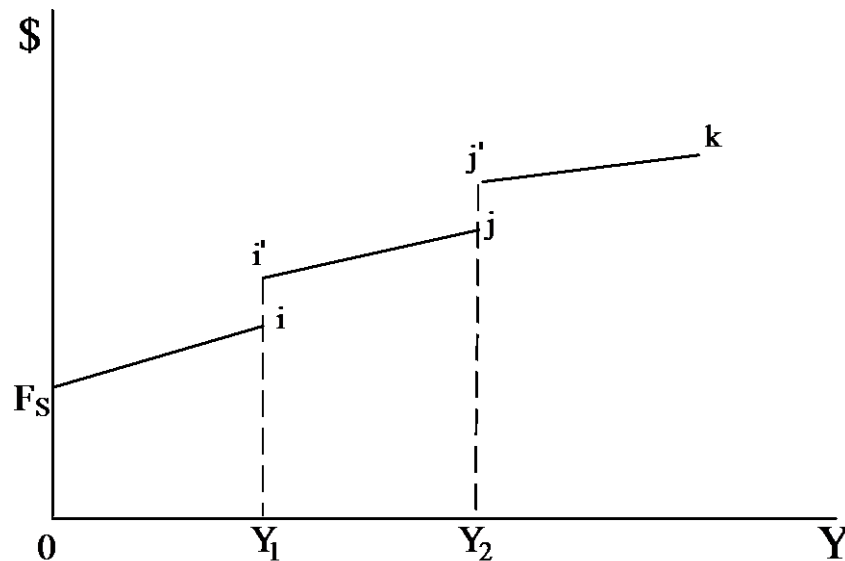
$$FOC_s = [1 + \{(L / L_B) - 1\}] B_B \cdot H_s f(Y_s) \quad (5)$$

where  $L$  is the carrier's labour price,  $L_B$  is the base carrier's labour price,  $B_B$  is base carrier's cost per block hour.

In addition, it is necessary establish a relationship between number of passengers on segment,  $Y_s$ , and the flight frequency,  $f_s$ . Since, in the long run, airlines attempt to achieve the optimal [profit maximizing] load factor it is possible to define the long run effective capacity ( $K_s$ ) for a given aircraft for a given segment. Therefore, the following relationship can be specified.

$$f_s (Y_s) = Y_s / K_s \quad (6)$$

In the figure below we have illustrated how costs might behave in some circumstances. First, note that there are fixed flight costs given by  $F_S$  which are composed of airport station costs plus some portion of airline system costs. Included in this would be the fixed costs of adding a flight. The cost segment  $F_S j$  shows a constant marginal cost of adding passengers to the route up to a level of passengers  $q_1$ . This is the point where the load factor exceeds the optimal load factor for the carrier and in order to accommodate more demand it adds flight(s).



The cost of adding a passenger,  $MC_y$ , given by the first equation below, is the sum of the direct costs (1st term on the right hand-side) and the indirect costs (second term on the right-hand side). The indirect costs are the added flight costs associated with the

marginal passenger. The cost of adding a flight is shown as a shift in the  $MC_Y$  function from  $F_{Si}$  to  $i'j$ . The indirect component of  $MC_Y$  will decrease as the number of passengers rises.

$$\begin{aligned} \frac{\partial C_s}{\partial Y} &= \frac{\partial C_s(Y)}{\partial Y} + \left[ \frac{\partial C}{\partial f} \frac{\partial f}{\partial Y} \right] \cdot Y_s \\ \frac{\partial C_s}{\partial f_s} &= \frac{\partial C(f)}{\partial f_s} \cdot Y_s \\ \frac{\partial \left( \frac{\partial C}{\partial Y} \right)}{\partial f} &= \frac{\partial C}{\partial Y \partial f} < 0 \end{aligned} \quad (7)$$

Although we can see this step function characterizing costs in the short term, in the long run equation (6) will be a fair description of cost behavior. Meaning firms will use management as well as investment techniques to utilize their 'effective' capacity which is profit maximizing load factor times aircraft seats.

The indirect and administration costs related to a particular segment can be computed via the following procedure.

(1) Collect data for total indirect cost (IC) for a set of U.S. airlines from Form 41 data and transform it in the following way:<sup>3</sup>

$$\begin{aligned} \text{IC} &= \text{total operating expenses} \\ &\quad - \text{flying operations costs} \\ &\quad - \text{maintenance costs} \\ &\quad - \text{depreciation and amortization} \end{aligned}$$

(2) Regress IC on the following variables:

$$\begin{aligned} \text{IC} &= I(Y, \text{RPK}, F, S, W, D) \\ &= a + c_1 Y + c_2 \text{RPK} + c_3 F + c_4 S + c_5 W \\ &\quad + c_{13} YF + c_{15} YW + c_{45} (S \cdot W) \\ &\quad + \sum c_{fi} D_i \end{aligned} \quad (8)$$

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<sup>3</sup> In this calculation the portions for freight, mail, and incidental outputs must be removed, perhaps, in proportion to their respective revenues.

where  $Y$  is firm's total number of passenger enplanements,  $RPK$  is total revenue-passenger-kilometers (or RPM),  $F$  is the total number of revenue flight departures performed,  $S$  is the number of route segments served,  $W$  is input price index, and  $D$  are firm dummy variables. We should see if we need firm dummy variables due to collinearity among variables, firm dummies tend to take away some portion of variations the economic variables can legitimately explain). In fact,  $c_4$  is the fixed cost associated with a route segment, that is  $A_s$  in equation (3).

(3) Evaluate the following expression in order to calculate incremental indirect cost of adding a route segment  $s$ :

$$\begin{aligned} \Delta IC &= I(Y + \Delta Y, RPK + \Delta RPK, F + \Delta F, S + 1, W, D) \\ &\quad - I(Y, RPK, F, S, W, D) \end{aligned} \quad (9)$$

The segment total cost function in (3) can now be obtained by adding equations (4) and (9).

$$\begin{aligned} C(Y_s, f_s) &= FOC_s + \Delta IC \\ &= [1 + \{(L / L_B) - 1\}] B_B H_s f(Y_s) + \Delta IC \end{aligned} \quad (10)$$

### **Ajustment for Productive Efficiency**

Since different airlines have different productive efficiency, it is necessary to adjust the segment costs for the productive efficiency differentials as well as the input cost differentials. We propose using the residual efficiency differences between major carriers identified in Oum and Yu (1995).<sup>4</sup> The same source will be used for input price differentials. In other words, the segment total cost in equation (8) will need to be adjusted upward or downward by the percentage of residual productivity differentials between the base carrier and the carrier of our concern.

### **(C) Computation of per-passenger cost on carrier-route combination $r$**

#### ***One-segment O-D case:***

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<sup>4</sup> See T. Oum and C. Yu, A Comparative Study of Productivity and Cost Competitiveness of the World's Major Airlines, (Discussion Paper 363 Institute of Social and Economic Research, Osaka University, 1995)

For direct route, computation is trivial. per-passenger marginal cost can be approximated by dividing the segment total cost by the number of passengers using segment  $r$ .

$$\frac{\partial C}{\partial Y_r} = \left( \Delta IC|_r + \left[ 1 + \left\{ \left( \frac{L}{L_B} \right) - 1 \right\} \right] \cdot B_B \cdot H_r \right) / K_r \quad (11)$$

### Multiple Segment O-D case:

For the case of multiple-segment O-D passengers, we need to add the per-passenger costs of all segments that the route  $r$  uses:

$$\frac{\partial C}{\partial Y_r} = \sum_{s \in r} \left\{ \left( \Delta IC|_r + \left[ 1 + \left\{ \left( \frac{L}{L_B} \right) - 1 \right\} \right] \cdot B_B \cdot H_r \right) / K_r \right\} \quad (12)$$

### (D) Entry and Exit of Carriers

Entry affects costs in two ways. First, entry of a low cost carrier affects incumbents' costs by putting pressure on input prices and productive efficiency. Second, it changes incumbents' passenger volume, which, in turn, changes their per-passenger segment costs by being at a different point on the economies of density curve.

In our model, we assume that an entry induces the incumbents to reduce their unit costs by a certain proportion,  $\theta$ , of the difference between each incumbent and the lowest cost entrant as described by the following formula:

$$C_i^n = C_i - \theta \{C_i - C_e\} \quad (11)$$

which are the unit costs  $C_e$  in equations 4, 9 and 10 and where  $C_i^n$  is the updated cost of carrier  $I$  after an entry,  $C_i$  is the cost prior to an entry, and  $C_e$  is the cost for the least cost entrant. In addition, a change in traffic density (passengers and flight frequency) will also affect unit costs of the incumbent carriers. Therefore, the changed values of  $Y$  and  $F$  should be plugged in segment total cost function (8).

### (E) Determination of Frequency

If there is no regulatory restriction on capacity, carriers will offer frequency until the marginal revenue from an additional flight equals its marginal cost.

Case of one-segment route with one fareclass

$$\text{Route's total revenue } R_r = q_r p_r$$

Marginal revenue from a flight is:

$$\frac{\partial R_r}{\partial f_r} = \frac{\partial q_r}{\partial f_r} p_r = \frac{\partial q_r}{\partial p_r^*} \frac{\partial p_r^*}{\partial f_r} p_r = (\beta \eta_r) \cdot \frac{q_r}{f} \quad (12)$$

where  $\eta_r$  is marshallian price elasticity of demand on route-carrier combination  $r$ ,  $\beta$  is the elasticity of hedonic price with respect to frequency  $f_r$ . In fact,  $\beta \cdot \eta_r$  is the frequency elasticity of the demand for carrier-route combination  $r$  (see, Oum, 1979, Bell J. of Economics for the derivation of this expression).

The Marginal Cost of a flight in segment  $S$  is computed as:

$$\frac{\partial C_s}{\partial f_s} = \frac{\partial C^y(\bullet)}{\partial f_s} + \frac{\partial C^y(\bullet)}{\partial Y_s} \frac{\partial Y_s}{\partial f_s} \quad (13)$$

In our case, the incremental cost of a flight for an incumbent carrier can be computed as:

$$\begin{aligned} MC_f &= \text{FOC for a flight} + \text{Incremental Indirect Cost} \\ &= \{ [ 1 + \{(L / L_B - 1)\} B_B H_r ] \\ &\quad + [ I ( Y + K, RPK + (K * \text{distance}), F + 1, S, W, D \\ &\quad - I (Y, RPK, F, S, W, D) ] \end{aligned} \quad (14)$$

The optimal flight frequency for segment  $s$  can then be determined by equating  $MR_f$  in equation (12) with  $MC_f$  in equation (13).

### Case of one-segment route with two classes

The Total revebnue from a route is calculated as  $R_r = q_{r1} p_{r1} + q_{r2} p_{r2}$ . The Marginal revenue from a flight calculated as:

The marginal cost of a flight can be computed as before.

$$\frac{\partial R_r}{\partial f_r} = \frac{\partial q_{r1}}{\partial f_r} \cdot p_{r1} + \frac{\partial q_{r2}}{\partial f_r} \cdot p_{r2} = \frac{\partial q_{r1}}{\partial p_{r1}^*} \frac{\partial p_{r1}^*}{\partial f_r} \cdot p_{r1} + \frac{\partial q_{r2}}{\partial p_{r2}^*} \frac{\partial p_{r2}^*}{\partial f_r} \cdot p_{r2} \quad (15)$$

### Case of a Segment with connecting traffic:

When a flight segment handles connecting traffic for various O-D passengers, the marginal revenue of a flight must include appropriate portions of the ticket prices these connecting passengers paid. Therefore, the incremental revenue from a flight can be computed as follows:

$$\frac{\partial R_s}{\partial f_s} = (\beta \eta_s) \cdot \frac{E_s}{f_s} + \sum \tau_i \frac{E_i}{f_s} \quad (16)$$

where the second term is summed over all O-D passengers travelling via segment s, and  $\tau_i$  is the fraction of ticket price for i th O-D that belong to flight segment s.

The marginal cost of an additional flight on segment s is given by:

$$\frac{\partial C}{\partial f_s} = [1 + \{(UL_B) - 1\}] B_B \cdot H_s + \Delta IC|_f \quad (17)$$

where  $\Delta IC|_f$  is computed as in equation (14).

### Markup

The markup which will be charged by a carrier in a given OD market will depend on the price elasticity of the demand market and the nature of competition between carriers. The nature of competition is measured by conjectural variations.

Past research has shown the following relationship between price, marginal cost and the behavior of firms under conditions of competition among few firms:

Past research has shown the following relationship between price, marginal cost and the behavior of firms under conditions of competition among few firms:

$$P_r^{OD} = \frac{MC_r^i \cdot \eta_{OD}}{\eta_{OD} - (1 + v_r^i) \cdot S_r^i} \quad (18)$$



- $\eta_{OD}$  - price elasticity of demand the O-D market
- $s_r^i$  - is the share of the O-D market by carrier i along route r
- $v_r^i$  - the conjecture variation of firm i
- $MC_r^i$  - the marginal cost per passenger of carrier i on route r

The conjectural variation is the carrier's expectation with regard to the anticipated reaction of competitors in the O-D market. The parameter  $v_r^i$  assumes the value of 1 when the firms collude with one another. The parameter is -1 when the firms view price as their strategic decision variable (Bertrand) and the parameter is 0 when the firms view quantity as their strategic decision variable (Cournot).

Changes in frequency (f) affects the price in two ways. First, it changes marginal cost through equation (3). Second, it changes market shares via changing hedonic price in equation 7 in the demand write-up.

### **Case of two-segment route: A to B and B to C**

In this case, three O-D traffics are being handled on the two route segments: AB (3rd/4th freedom segment), BC (5th freedom segment), and AC (connecting service routes). Therefore, the revenue function for this route r for the case of two fare classes can be written as:

$$R_r = (q_1^{AB} p_1^{AB} + q_2^{AB} p_2^{AB}) + (q_1^{BC} p_1^{BC} + q_2^{BC} p_2^{BC}) + (q_1^{AC} p_1^{AC} + q_2^{AC} p_2^{AC}) \quad (19)$$

Since 5th freedom segment (BC) usually has a lower frequency than 3rd/4th freedom segments, the frequency available for O-D traffic AC is:

$$f^{AC} = \min \{ f^{AB}, f^{BC} \} = f^{BC} \quad \text{Because } f^{AB} \geq f^{BC} \quad (20)$$

An additional flight on segment BC allows the carrier to sell additional tickets in both BC and AC markets. Therefore, marginal revenue from an additional flight on segment BC can be calculated as:

$$R_f^{BC} = (q_1^{BC} p_1^{BC} + q_2^{BC} p_2^{BC}) + \cdot \cdot (q_1^{AC} p_1^{AC} + q_2^{AC} p_2^{AC}) \quad (21)$$

where  $\cdot$  is fraction of the revenue from AC market to be allocated to flight segment BC.

$$\frac{\partial R_f^{BC}}{\partial f^{BC}} = \frac{\partial q_1^{BC}}{\partial p_1^{*BC}} \frac{\partial p_1^{*BC}}{\partial f_{BC}} p_1^{BC} + \frac{\partial q_2^{BC}}{\partial p_2^{*BC}} \frac{\partial p_2^{*BC}}{\partial f_{BC}} p_2^{BC} + \tau \cdot \left[ \frac{\partial q_1^{AC}}{\partial p_1^{*AC}} \frac{\partial p_1^{*AC}}{\partial f_{AC}} p_1^{AC} + \frac{\partial q_2^{AC}}{\partial p_2^{*AC}} \frac{\partial p_2^{*AC}}{\partial f_{AC}} p_2^{AC} \right] \quad 10(22)$$

The optimal frequency is determined by equating the above marginal revenue of an additional flight to the marginal cost of a flight for flight segment BC computed similarly as in (18). When flight frequency is restricted by bilateral agreement and the unconstrained optimal frequency computed above is larger than the allowed frequency, then the allowed frequency becomes the carrier's frequency.